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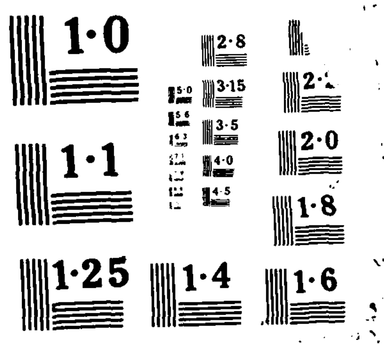
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# TOKEN PERFORMANCE AND LOCAL AREA NETWORKS

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* Supported by U. S. Army Research Contract No. DAAG29-85-G-0109

# NUMERICAL SOLUTION OF A SYSTEM OF RANDOM VOLTERRA INTEGRAL EQUATIONS I: SUCCESSIVE APPROXIMATION METHOD*

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## Abstract

In this article we discuss successive approximation method for a system of random Volterra integral equations. An example is presented to implement the theory. Kolmogorov-Smirnov test is used to fit a distribution of the solutions.

## 1. Introduction

The study of random Volterra integral equations and their applications play an important role in the area of probabilistic analysis. One of the main reasons is that integral equations are suitable for numerical treatment. For a recent survey of approximate solution of random integral equations we refer to Bharucha-Reid and Christensen [4]. For the numerical treatment of random integral equations we refer to Bharucha-Reid [3], Becus [2], Christensen and Bharucha-Reid [5], Lax [8-10], Tsokes and Padgett [14]. For a detailed survey of analytical and numerical methods

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To appear in Conference Volume of the International Conference in Differential Equations, University of Alabama, Birmingham, AL.

# APPROXIMATE SOLUTION OF RANDOM DIFFERENTIAL EQUATION*

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## Abstract

Chebyshev method for solving random differential equation is presented. The convergence of the random coefficients of the Chebyshev series is established. Statistical properties of the random coefficients are discussed.

## 1. INTRODUCTION

In recent years, increasing interest in the numerical solution of random differential equations has led to the progressive development of several numerical methods. A large number of papers have appeared in the literature containing approximate solutions of random differential equations. For Newton's method, successive approximations, perturbation methods, method of moments, finite element methods and other methods on the approximate solution of random equations we refer to Bharucha-Reid [1]. Numerical methods of random polynomials can be found in Bharucha-Reid and Sambandham [2]. For a short and elegant note on several of these methods we also refer to Lax [9, 10]. Some analytical and numerical estimates on error estimates of stochastic differential equations are presented in Ladde et al. [6-8]. For other interesting numerical techniques we refer to Boyce [3] and [11]. Numerical treatment of Ito equations can be found in Klauder and Petersen [4], Rumelin [12] and Taley [15]. We notice that most of these numerical methods are successful

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# *A Mixed Min-Max Control Problem Governed by Integral Equations

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A mixed min-max control problem is considered using relaxed controls. Mixed min-max problems governed by differential equations have been considered using Dubovitskii and Milyutin Theory and Convex Analysis [6]. Here we use relaxed controls and penalization to deal with a process governed by integral equations. We use a technique employed by us in dealing with control problems governed by integral equations [2].

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# NUMERICAL SOLUTIONS TO STOCHASTIC DIFFERENCE EQUATIONS*

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ABSTRACT

Statistical properties of the numerical solutions of random difference equations are estimated. By an application of variation of constant formula, error estimates between random solutions and smooth solutions (deterministic solution) are discussed.

## 1. INTRODUCTION

Numerical solutions of mathematical models of dynamical systems in applied mathematics demand a fair knowledge of stochastic difference equations. That is stochastic difference equations where some of the variables can change stochastically in time. More recently Ladde and Sambandham [5] developed several random difference inequalities which are very powerful tools to study stability properties of stochastic difference equations. By an application of variation of constant formula, error estimates between stochastic systems and the respective deterministic (smooth) systems are discussed in [6]. These estimates provide statistical properties of the upper bound for the error estimates. For related results we refer to Fai Ma and Caughey [2,3,4], Mann and Wald [7], Deller [1].

We organize our article as follows. In Section 2 we present the analytical upper bounds for error estimates. In Section 3 we include a few discussions on the numerical solutions. In addition, we have presented a few tables and figures to illustrate the behavior of the error between the mean of the random solutions and deterministic solutions. We

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NUMERICAL SOLUTION TO A SYSTEM OF
RANDOM VOLTERRA INTEGRAL EQUATIONS*

N. Medhin[1], M. Sambandham[2] and C. K. Zoltani[3]

Abstract

In this article we present a brief summary of the numerical solution of a system of random Volterra integral equations. The methods we use are (i) Newton's method and (ii) successive approximation method. Based on the simulation, we discuss the mean and variance of the solution of a system of random Volterra integral equations.

*Supported by U.S. Army Research Contract No. DAAG29-85-G-0109.

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GAS FLOWS GENERATED BY SOLID-PROPELLANT BURNING UNDER RANDOM ENVIRONMENT.

In the literature it is customary to assume that the local mass source term, momentum source term and energy source term to be deterministic. In this paper we consider some of these source terms to be random. We assume that the flow generated by gasification of the solid propellant material is one dimensional and unsteady. By an application of random choice method, we discuss the statistical properties of all field variables.

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Deadline for Submission of Abstract
NOVEMBER 28th, 1986

(PLEASE COMPLETE PAGE 2)

# Optimal Processes Governed by Integral Equations with Unilateral Constraint

Negash G. Medhin*

An optimal process with unilateral constraint is considered using relaxed controls. Existence of optimal control is automatic. Further, problems with linear and certain quadratic cost functions with linear integral and state constraints are such that the controls are actually ordinary, i.e., bounded measurable functions. In addition, if the control set is a convex polyhedron the optimal controls take their values in the vertices of the polyhedron. Control processes governed by integral equations have been considered in [2]. However, in [2] existence is not shown for the general problem and optimality conditions are not available for the problem when state constraints are present. Furthermore, the work in [2] has some errors as pointed out in [3]. Finally we point out the basic idea in our approach in this paper is the same as in [6].

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# *A Mixed Min-Max Control Problem Governed by Integral Equations

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A mixed min-max control problem is considered using relaxed controls. Mixed min-max problems governed by differential equations have been considered using Dubovitskii and Milyutin Theory and Convex Analysis [6]. Here we use relaxed controls and penalization to deal with a process governed by integral equations. We use a technique employed by us in dealing with control problems governed by integral equations [2].

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# Mixed Min-Max Optimization Problem with Restrictions *

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## Introduction

We consider a min-max optimization problem using relaxed controls. Such problems have been considered using Dubovitskii and Milyutin Theory and convex analysis [5]. We reformulate the problem where a unilateral constraint is added. We deal with the reformulated problem using penalization [2], [3]. Our procedure produces more optimality conditions than [5] and also more insight into [5]. In the special case where the controls appear linearly and the control sets are convex polyhedra the relaxed controls are ordinary controls taking values in the vertices of the polyhedra. Finally, we present examples where we demonstrate how the optimal controls and trajectories could be calculated and some that are numerically worked out.

KEY WORDS: Relaxed controls, unilateral constraint, penalization

* Supported by U.S. Army Research Contract No. DAAG29-85-G-0109.

# NUMERICAL SOLUTION TO A SYSTEM OF RANDOM VOLTERRA INTEGRAL EQUATIONS*

N. Medhin[1], M. Sambandham[2] and C. K. Zoltani[3]

Abstract

In this article we present a brief summary of the numerical solution of a system of random Volterra integral equations. The methods we use are (i) Newton's method and (ii) successive approximation method. Based on the simulation, we discuss the mean and variance of the solution of a system of random Volterra integral equations.

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APPROXIMATE SOLUTION OF RANDOM

DIFFERENTIAL EQUATION*

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Abstract

Chebyshev method for solving random differential equation is presented. The convergence of the random coefficients of the Chebyshev series is established. Statistical properties of the random coefficients are discussed.

## 1. INTRODUCTION

In recent years, increasing interest in the numerical solution of random differential equations has led to the progressive development of several numerical methods. A large number of papers have appeared in the literature containing approximate solutions of random differential equations. For Newton's method, successive approximations, perturbation methods, method of moments, finite element methods and other methods on the approximate solution of random equations we refer to Bharucha-Reid [1]. Numerical methods of random polynomials can be found in Bharucha-Reid and Sambandham [2]. For a short and elegant note on several of these methods we also refer to Lax [9, 10]. Some analytical and numerical estimates on error estimates of stochastic differential equations are presented in Ladde et al. [6-8]. For other interesting numerical techniques we refer to Boyce [3] and [11]. Numerical treatment of Ito equations can be found in Klauder and Petersen [4], Rumelin [12] and Taley [15]. We notice that most of these numerical methods are successful

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# A METHOD FOR THE NUMERICAL SOLUTION OF RANDOM CAUCHY SINGULAR INTEGRAL EQUATION

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ABSTRACT

The solution of several elasticity problems, and particularly crack problems, can be reduced to the solution of one-dimensional singular integral equations with a Cauchy-type kernel. In this paper, we present a method for the numerical solution of a random singular integral equation of the Cauchy-type. To illustrate this method, it is applied to the random singular integral equation that arises in the problem of periodic array of straight cracks inside an infinite isotropic elastic medium and subjected to random pressure distribution along the crack edges. The statistical properties of the random solution are evaluated numerically, and used to determine the stress intensity factor at the crack tips. Results of this study highlight the advantage of using this method to solve a random Cauchy-type singular

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integral equation. The method is applied to the crack problem subjected to different forcing functions.

## 1. INTRODUCTION

In recent years, an unprecedented widespread interest in the numerical solution of singular integral equations of the Cauchy-type has resulted in the development of several numerical methods [1-5] besides stimulating a mathematical interest in their numerical analysis. Since the pioneering work by Erdogan [1] in 1969 and subsequently by Erdogan and co-workers [3], a large number of papers have appeared in the literature concerning singular integral equations that arise in several fields of mathematical physics and engineering. The solution methods developed have aimed at reducing the singular integral equation to a system of linear algebraic equations which can be easily solved to give an approximate expression for the solution of a singular integral equation or solutions in the case of a system of singular integral equations.

Several of the numerical methods developed over the years were found to have disadvantages due to their inherent complexity, low degree of accuracy and limitations as regards the classes of singular integral equations to which they were applicable. It is known that numerical solutions of Fredholm integral equations by reduction to a system of linear equations could be extended to the case of Cauchy singular integral equations (CSIEs), that is to the case when the kernels of the Fredholm integral equations have Cauchy-type singularities. Since Cauchy-type singular integral equations on closed contours have been treated elsewhere [7, 8], in this paper we confine our attention to equations on intervals (a,b) of the real line, which may be either finite or infinite. In particular, we consider linear equations since the theory for these is fairly complete and well developed. The most frequently used technique for the numerical solution of linear singular integral equations consists in approximating the integral terms by using an approximate numerical integration rule and applying the singular integral equation at appropriately selected collocation points [2-4, 9]. One determination of the unknown function at the abscissas used, the function is approximated along the whole integration interval [10 ].

A wide body of literature pertaining to the numerical solution of singular integral equations of the deterministic type is succintly summarized by Ergodan, et al. [3], Golberg [11], and Theocaris [6]. In referring to singular integral equations, Erdogan, Golberg and Theocaris in their articles imply Cauchy-type singular integrals and Cauchy-type singular integral equations. We refer to Bharucha-Reid and Bharucha-Reid and Christensen [12, 13] for surveys of the different analytical methods for the solution of random integral equations. Most recently, Christensen and Bharucha-Reid [14] developed numerical solution procedures for Fredholm equations with random kernels, random right hand side or both and for Fredholm equations with random degenerate kernels [15]. The Cauchy-type singular integral equations which arise frequently in the fields of aerodynamics, hydrodynamics, solid mechanics, elasticity and fracture mechanics, are closely related to Fredholm-type equations. There exists a pragmatic need to develop numerical techniques in order to be able to study the characteristics of the random singular integral equations. The rationale for this study was to extend the concept for the solution of random Fredholm integral equations [14-16], to the solution of random singular integral equations of the Cauchy-type that arise in crack problems in the classical theory of elasticity. In particular, we consider the problem of an array of periodic cracks inside an infinite isotropic elastic medium and subjected to random pressure distribution along the crack edges, i.e., the case of a random forcing function. The statistical properties of the solution are evaluated and the results obtained for the random equation are compared with those of the deterministic equation. Further we also discuss the distribution of the stress intensity factor for different sample sizes.

## 2. SINGULAR INTEGRAL EQUATIONS

Consider the singular integral equation of the form

$$\frac{1}{\pi}\int_{-1}^{1}\frac{\phi(t)}{t-x}\,dt+\int_{-1}^{1}k(x,t)\phi(t)\,dt=f(x),\qquad -1<x<1, \tag{2.1}$$

where $k(x,t)$ is a regular kernel. $\phi(t)$ is the unknown function proportional to the crack tip opening displacement and $f(x)$ is the known function representing pressure distribution along the crack.

Let $g(t) = \phi(t)/z(t)$, where $z(t) = (1-t^2)^{-1/2}$ and

$$(2.2) \qquad \int_{-1}^{1} \phi(t)dt = 0.$$

We consider (2.1) together with (2.2) and replace $g(t)$ by a new unknown function $G(x)$ defined by the first integral of (2.1), that is,

$$(2.3) \qquad G(x) = \frac{1}{\pi} \int_{-1}^{1} (1-t^2)^{-1/2} \frac{g(t)}{t-x} dx, \qquad -1 < x < 1.$$

Since $g(t)$ satisfies (2.2), we can express $g(t)$ in terms of $G(x)$ by

$$(2.4) \qquad g(t) = \frac{1}{\pi} \int_{-1}^{1} (1-y^2)^{1/2} \frac{G(y)}{t-y} dy, \qquad -1 \le t \le 1.$$

By an application of (2.3) and (2.4), we can write (2.1) in the form

$$(2.5) \qquad G(x) + \int_{-1}^{1} (1-y^2)^{1/2} K(y,x)G(y)dy = f(x) \qquad -1 < x < 1,$$

where

$$(2.6) \qquad K(y,x) = \frac{1}{\pi} \int_{-1}^{1} (1-t^2)^{-1/2} \frac{K(t,x)}{t-y} dt$$

We remark that (2.5) is another form of (2.1). The equation (2.5) is Fredholm equation of second type and its numerical solution can be found by quadrature method.

The probabilistic analogue of (2.5) is the random equation. We call an equation random if certain components such as coefficients, kernels, nonhomogeneous terms or forcing functions and initial and or boundary data are random functions. The random version of (2.5) is of the form

$$(2.7)\qquad G(x,\omega) + \int_{-1}^{1} (1-y^2)^{1/2} K(y,x,\omega) G(x,\omega)\,dy = f(x,\omega) \qquad -1 < x < 1$$

where $K(y,x,\omega)$ is the random kernel and $f(x,\omega)$ is the random forcing term. The parameter $\omega$ is an element of a given probability space $(\Omega, A, P)$. Here $G(x,\omega)$ is the random function referred to as the solution or output. For numerical solution of (2.7) we refer to [14, 15, 16].

In this article we consider a periodic array of cracks along a straight line in an infinite isotropic elastic medium under plane strain or generalized plane stress. The length of each crack is assumed to be '2a' and the period of the array is 'b'. For the case of a constant compressive loading distribution along the crack edges, Ioakimidis [17] found the singular integral equation to be

$$(2.8)\qquad \frac{a}{b} \int_{-1}^{1} (1-t^2)^{-1/2} \cot\left(\frac{\pi a(t-x)}{b}\right) g(t)\,dt = f(x) \qquad -1 < x < 1$$

which is of the form (2.1) with

$$(2.9)\qquad k(t,x) = \frac{a}{b} \cot\left(\frac{\pi a(t-x)}{b}\right) - \frac{1}{\pi(t-x)},$$

where cracks are assumed to be loaded by constant compressive loading distribution $f(x)$ along both crack edges. If the loading distribution $f(x)$ is random, then (2.8) is a random singular integral equation. If we denote random loading distribution by $f(x,\omega)$ then the random version of (2.8) is

$$(2.10)\quad \frac{a}{b}\int_{-1}^{1}(1-t^2)^{-1/2}\cot\left(\frac{\pi a(t-x)}{b}\right)g(t,\omega)dt = f(x,\omega) \quad -1 < x < 1$$

Equation (2.10) together with (2.2) can be solved numerically by methods suggested in [12-17].

## 3. NUMERICAL TREATMENT

In this section we discuss the numerical solution of (2.10). We assume the ratio $a/b = .4$ and the successive cracks lie close to each other. We denote the value of the stress intensity factor by $K(\pm 1,\omega)$ at the crack tips $t = \pm 1$. It is given by

$$(3.1)\qquad K(\pm 1,\omega) = \pm g(\pm 1,\omega)$$
$$\approx \frac{1}{N}\sum_{k=1}^{N}(1 \pm x_k)G(x_k,\omega).$$

where $x_k$ are suitable nodes

$$x_k = \cos(k-0.5)\frac{\pi}{N}$$

which we use to solve (2.7). The unknown random function $g(x,\omega)$ in (2.10) is evaluated at different pressure distributions along the crack edges, namely,

<u>Example 3.1</u>. $f(x,\omega) \in N(1,\sigma^2)$

Example 3.2. $f(x,\omega) \in N(e^x,\sigma^2)$

Example 3.3. $f(x,\omega) \in N(e^x,\sigma^2 e^{2x})$

A sample of 5000 standard normal random numbers are generated from IMSL routines (GGNML). The resulting random numbers are used to generate the random input functions, namely, the random pressure distributions in the manner described above. For each random input function, equation (2.7) and hence equation (2.10) is solved numerically to obtain the random output function, and the stress intensity factor. At the crack tips, $K(\pm 1,\omega)$ are computed based on the relationship in equation (3.1).

## 4. RESULTS AND DISCUSSION

Accurate estimates of the stress intensity factor at the crack tips under more realistic pressure distributions is essential for the reliable prediction of fatigue crack growth rates in structural members. Several analytical techniques are presently available to obtain solutions to random Cauchy-type singular integral equations that arise in crack problems. Most of these numerical techniques for obtaining the stress intensity factors for crack and elasticity problems have been concerned with pressure distributions that are purely deterministic in nature.

In dealing with random equations, the main objectives to be concerned with are: (1) determination of the statistical properties of the random solution such as its expectation, variance and higher moments; (2) to establish the relationship between the expected solution of the random equation and the solution of the deterministic equation or the mean solution [18]; (3) determining the distribution of the solutions, and (4) discussing the limiting properties of the solutions. Of particular interest is the relationship between the statistical properties of the solution and the statistical properties of the random input function introduced into the equation. Table I-V summarize the statistical properties of $K(\pm 1,\omega)$.

The distribution of stress intensity factor $K(\pm 1,\omega)$ for the functions $f(x,\omega)$ are shown in Figures 1-30. These figures show that as the value of N increases, the distribution of $K(\pm 1,\omega)$ is approximately normal with $E[K(\pm 1,\omega)]$ being the solution of the mean equation. It is observed that as the value of N increases, the distribution of the stress intensity factor $K(\pm 1,\omega)$ is normal as long as the forcing function $f(x,\omega)$ is normal.

## 5. CONCLUSIONS

1. The foregoing analysis demonstrates a fairly simple yet useful method for the solution of random Cauchy-type singular integral equation that arises in a crack problem in the classical theory of elasticity. The solution of the random equation helps in the accurate evaluation of the stress intensity factor at the crack

tips under more realistic input or forcing functions.

2. The distributions of the stress intensity factors $K(\pm 1, \omega)$ are approximately normal when the forcing functions are normal.

3. The results of this study highlight the influence of the nature of the random input function on the stress intensity factor.

TABLE I

| $f(x,\omega) \in N(1,0.02^2)$ | | | |
|---|---|---|---|
| Sample Size | N | Mean of $K(1,\omega)$ | Variance of $K(1,\omega)$ |
| 500 | 2 | 1.634664 | 0.001065 |
| | 3 | 1.575876 | 0.000896 |
| | 5 | 1.563470 | 0.000866 |
| | 7 | 1.566512 | 0.000969 |
| | 9 | 1.566098 | 0.000910 |
| 1000 | 2 | 1.635569 | 0.001079 |
| | 3 | 1.575410 | 0.000960 |
| | 5 | 1.563799 | 0.000931 |
| | 7 | 1.567005 | 0.001003 |
| | 9 | 1.565704 | 0.000940 |
| 2000 | 2 | 1.636609 | 0.001098 |
| | 3 | 1.575890 | 0.000962 |
| | 5 | 1.564721 | 0.000965 |
| | 7 | 1.566515 | 0.000993 |
| | 9 | 1.564965 | 0.000987 |
| 3000 | 2 | 1.637003 | 0.001088 |
| | 3 | 1.575030 | 0.000962 |
| | 5 | 1.564731 | 0.000977 |
| | 7 | 1.566254 | 0.000990 |
| | 9 | 1.564981 | 0.000974 |
| 4000 | 2 | 1.636695 | 0.001094 |
| | 3 | 1.575152 | 0.000984 |
| | 5 | 1.565015 | 0.000977 |
| | 7 | 1.566710 | 0.000985 |
| | 9 | 1.564917 | 0.000983 |
| 5000 | 2 | 1.636726 | 0.001089 |
| | 3 | 1.575127 | 0.000969 |
| | 5 | 1.564924 | 0.000989 |
| | 7 | 1.566634 | 0.000969 |
| | 9 | 1.564916 | 0.000983 |

TABLE II

$f(x,\omega) \in N(e^x, 0.02^2)$

| Sample Size | N | Mean of $K(1,\omega)$ | Variance of $K(1,\omega)$ |
|---|---|---|---|
| 500 | 2 | 2.567233 | 0.000985 |
| | 3 | 2.548983 | 0.000967 |
| | 5 | 2.538348 | 0.000880 |
| | 7 | 2.539721 | 0.000997 |
| | 9 | 2.539926 | 0.001007 |
| 1000 | 2 | 2.566281 | 0.001026 |
| | 3 | 2.548836 | 0.001010 |
| | 5 | 2.537966 | 0.000897 |
| | 7 | 2.540643 | 0.000933 |
| | 9 | 2.539931 | 0.000971 |
| 2000 | 2 | 2.566361 | 0.001063 |
| | 3 | 2.548886 | 0.001033 |
| | 5 | 2.538958 | 0.000955 |
| | 7 | 2.540021 | 0.000937 |
| | 9 | 2.539534 | 0.000992 |
| 3000 | 2 | 2.566199 | 0.001082 |
| | 3 | 2.549063 | 0.001023 |
| | 5 | 2.539601 | 0.000987 |
| | 7 | 2.539973 | 0.000944 |
| | 9 | 2.539349 | 0.000992 |
| 4000 | 2 | 2.565732 | 0.001077 |
| | 3 | 2.548748 | 0.001026 |
| | 5 | 2.539587 | 0.000976 |
| | 7 | 2.539796 | 0.000949 |
| | 9 | 2.539454 | 0.001007 |
| 5000 | 2 | 2.565745 | 0.001069 |
| | 3 | 2.548838 | 0.001015 |
| | 5 | 2.539710 | 0.000975 |
| | 7 | 2.539714 | 0.000955 |
| | 9 | 2.539338 | 0.001008 |

TABLE III

| $f(x,\omega) \in N(e^x, 0.02^2 e^{2x})$ | | | |
|---|---|---|---|
| Sample Size | N | Mean of $K(1,\omega)$ | Variance of $K(1,\omega)$ |
| 500 | 2 | 2.565219 | 0.002773 |
| | 3 | 2.544691 | 0.002617 |
| | 5 | 2.539554 | 0.002960 |
| | 7 | 2.541145 | 0.002661 |
| | 7 | 2.542362 | 0.002784 |
| 1000 | 2 | 2.565728 | 0.002727 |
| | 3 | 2.546564 | 0.002680 |
| | 5 | 2.541254 | 0.002728 |
| | 7 | 2.541461 | 0.002704 |
| | 9 | 2.542656 | 0.002535 |
| 2000 | 2 | 2.565767 | 0.002784 |
| | 3 | 2.548370 | 0.002618 |
| | 5 | 2.539482 | 0.002647 |
| | 7 | 2.540158 | 0.002643 |
| | 9 | 2.540600 | 0.002528 |
| 3000 | 2 | 2.565145 | 0.002694 |
| | 3 | 2.547493 | 0.002589 |
| | 5 | 2.539537 | 0.002630 |
| | 7 | 2.539658 | 0.002603 |
| | 9 | 2.541302 | 0.002609 |
| 4000 | 2 | 2.565149 | 0.002677 |
| | 3 | 2.547726 | 0.002611 |
| | 5 | 2.539973 | 0.002641 |
| | 7 | 2.538959 | 0.002602 |
| | 9 | 2.540734 | 0.002614 |
| 5000 | 2 | 2.564995 | 0.002644 |
| | 3 | 2.547745 | 0.002592 |
| | 5 | 2.539966 | 0.002628 |
| | 7 | 2.539020 | 0.002585 |
| | 9 | 2.540420 | 0.002630 |

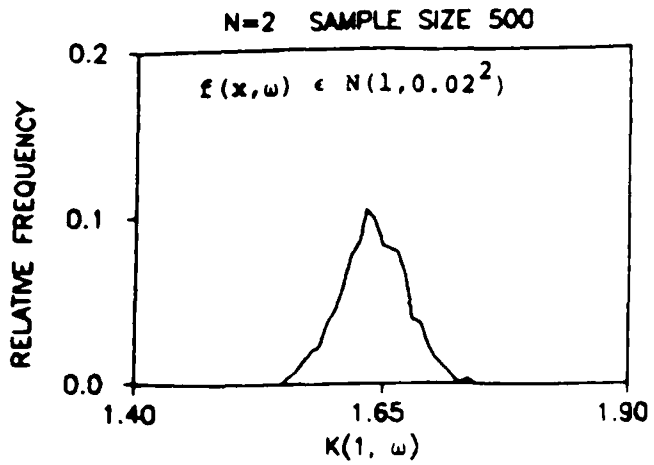
TABLE IV

| $f(x,\omega) \in N(e^{-x},0.02^2)$ | | | |
|---|---|---|---|
| Sample Size | N | Mean of $K(-1,\omega)$ | Variance of $K(-1,\omega)$ |
| 500 | 2 | 1.562386 | 0.000996 |
| | 3 | 1.478523 | 0.001107 |
| | 5 | 1.465161 | 0.000979 |
| | 7 | 1.466776 | 0.001004 |
| | 9 | 1.469071 | 0.000901 |
| 1000 | 2 | 1.563262 | 0.001020 |
| | 3 | 1.479762 | 0.001065 |
| | 5 | 1.465880 | 0.001015 |
| | 7 | 1.466557 | 0.000990 |
| | 9 | 1.468161 | 0.000939 |
| 2000 | 2 | 1.563492 | 0.001015 |
| | 3 | 1.479095 | 0.000998 |
| | 5 | 1.466543 | 0.001003 |
| | 7 | 1.466125 | 0.000990 |
| | 9 | 1.468237 | 0.000972 |
| 3000 | 2 | 1.562768 | 0.001029 |
| | 3 | 1.478868 | 0.000993 |
| | 5 | 1.467130 | 0.000992 |
| | 7 | 1.466727 | 0.000992 |
| | 9 | 1.468084 | 0.000990 |
| 4000 | 2 | 1.562527 | 0.001035 |
| | 3 | 1.478661 | 0.000985 |
| | 5 | 1.467698 | 0.000993 |
| | 7 | 1.466606 | 0.000992 |
| | 9 | 1.467634 | 0.000986 |
| 5000 | 2 | 1.562371 | 0.001046 |
| | 3 | 1.478330 | 0.000980 |
| | 5 | 1.467851 | 0.000987 |
| | 7 | 1.466590 | 0.000985 |
| | 9 | 1.467648 | 0.000978 |

TABLE V

$f(x,\omega) \sim N(e^{-x}, 0.02^2 e^{-2x})$

| Sample Size | N | Mean of $K(-1,\omega)$ | Variance of $K(-1,\omega)$ |
|---|---|---|---|
| 500 | 2 | 1.563375 | 0.000957 |
| | 3 | 1.480107 | 0.000776 |
| | 5 | 1.468341 | 0.000749 |
| | 7 | 1.467878 | 0.000831 |
| | 9 | 1.467650 | 0.000818 |
| 1000 | 2 | 1.562688 | 0.000943 |
| | 3 | 1.479829 | 0.000833 |
| | 5 | 1.467521 | 0.000833 |
| | 7 | 1.466508 | 0.000835 |
| | 9 | 1.468472 | 0.000826 |
| 2000 | 2 | 1.561791 | 0.000963 |
| | 3 | 1.479141 | 0.000852 |
| | 5 | 1.467389 | 0.000858 |
| | 7 | 1.467030 | 0.000867 |
| | 9 | 1.467370 | 0.000843 |
| 3000 | 2 | 1.561813 | 0.000973 |
| | 3 | 1.478927 | 0.000861 |
| | 5 | 1.467764 | 0.000881 |
| | 7 | 1.466812 | 0.000870 |
| | 9 | 1.467162 | 0.000861 |
| 4000 | 2 | 1.562072 | 0.000976 |
| | 3 | 1.478724 | 0.000876 |
| | 5 | 1.467863 | 0.000880 |
| | 7 | 1.467345 | 0.000872 |
| | 9 | 1.467029 | 0.000851 |
| 5000 | [illegible] | 1.561947 | 0.000975 |
| | [illegible] | 1.478948 | 0.000869 |
| | | 1.467711 | 0.000880 |
| | | 1.467042 | 0.000875 |
| | | 1.466858 | 0.000842 |

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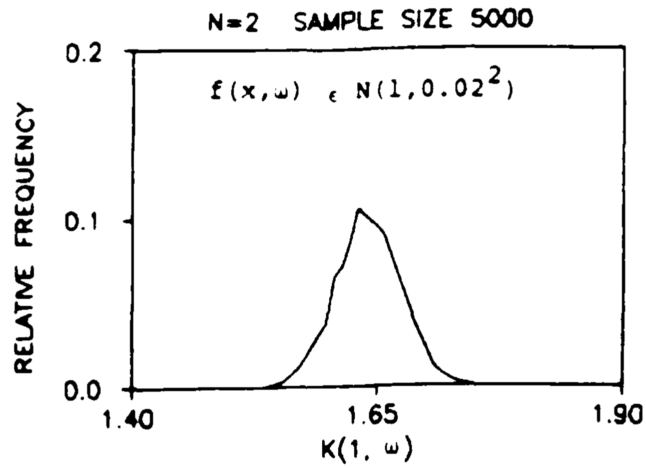


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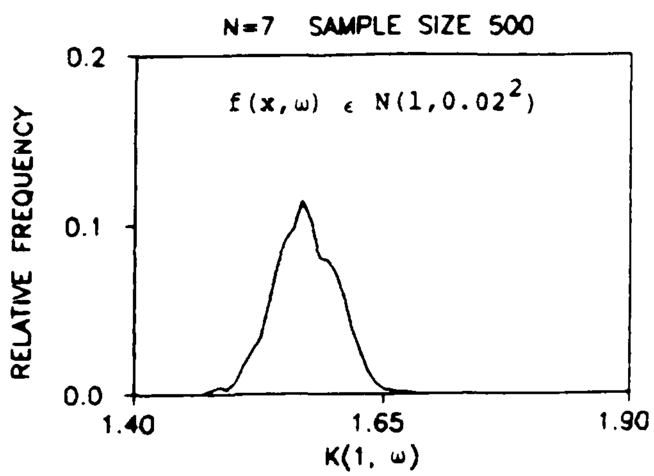


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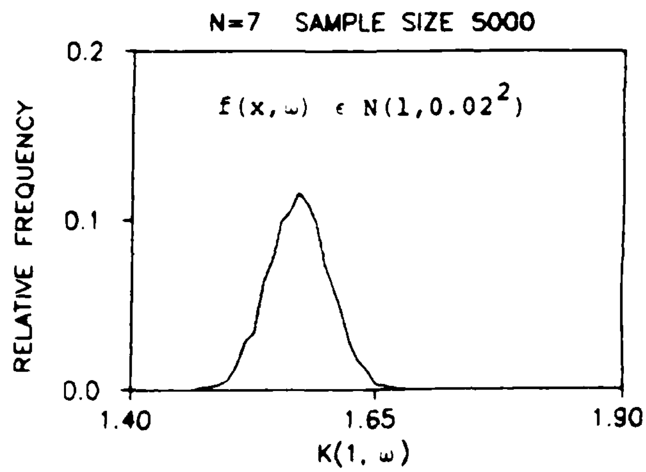


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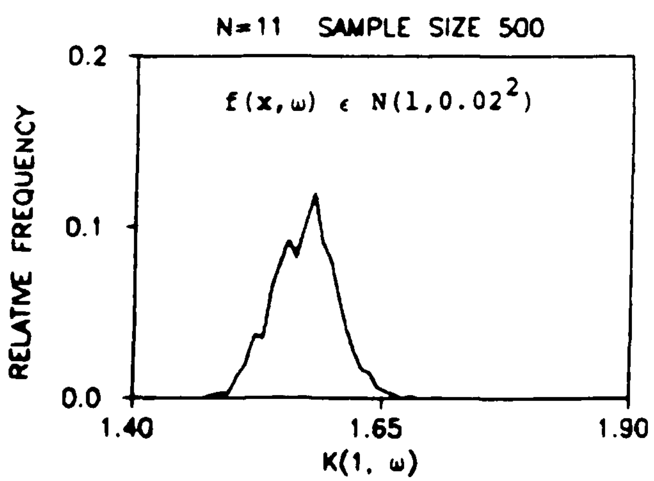


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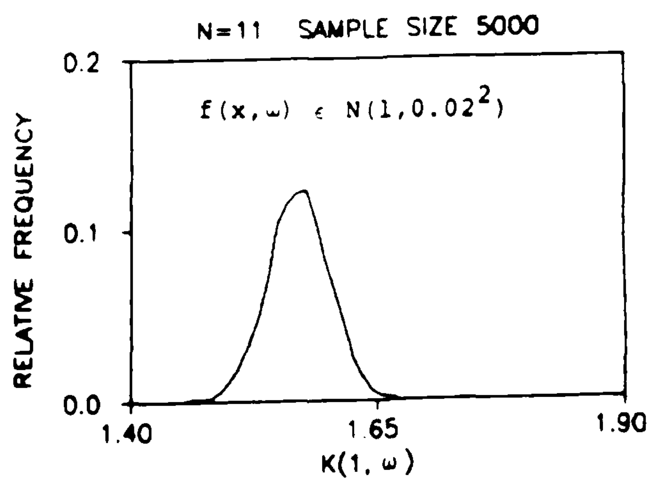


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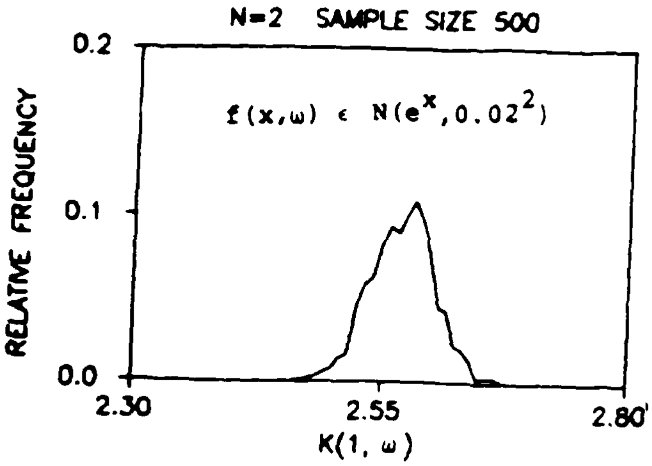


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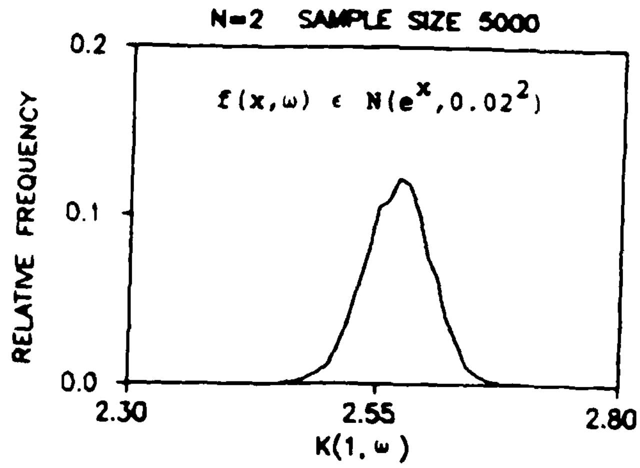


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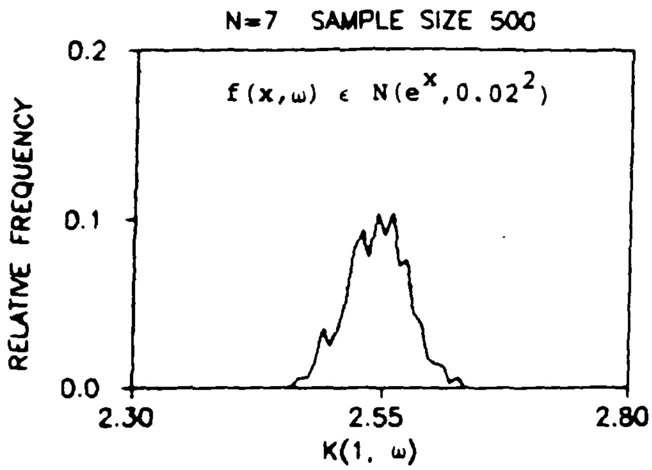


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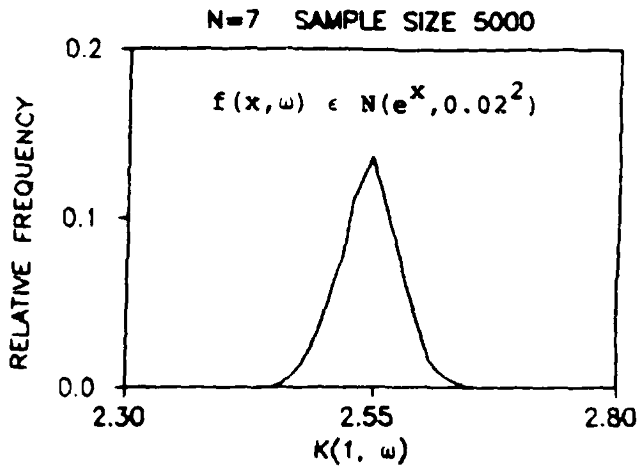


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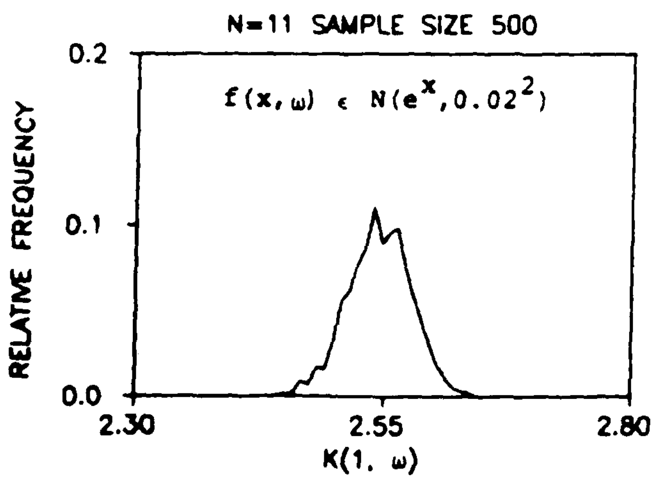


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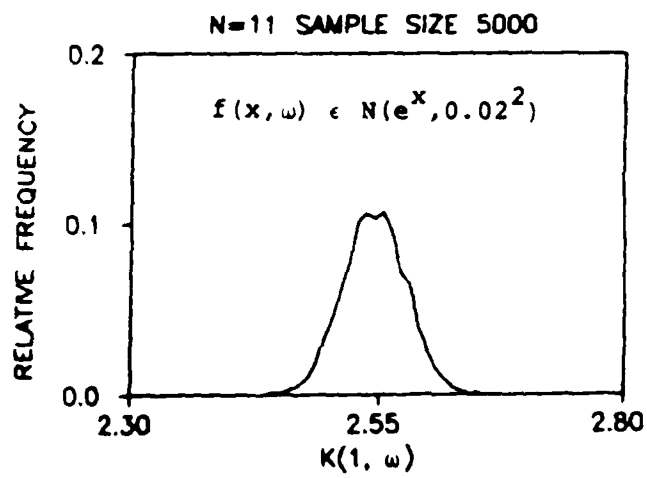


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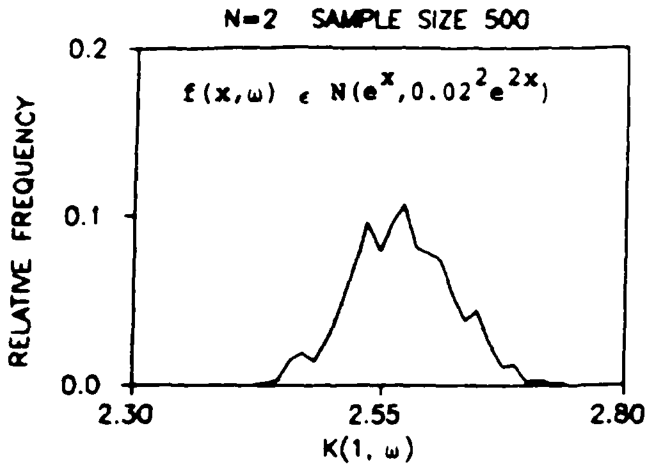


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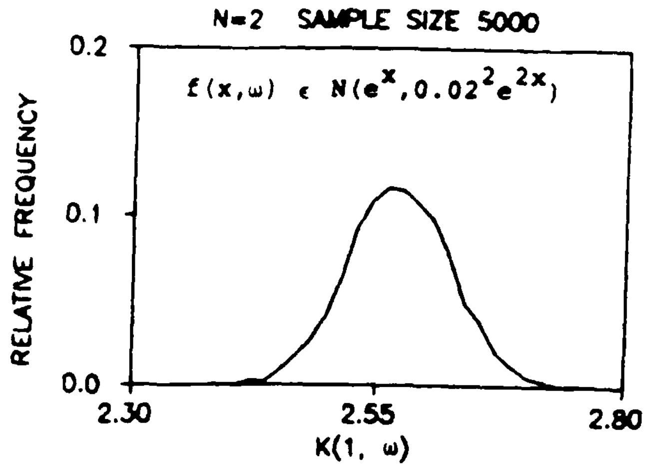


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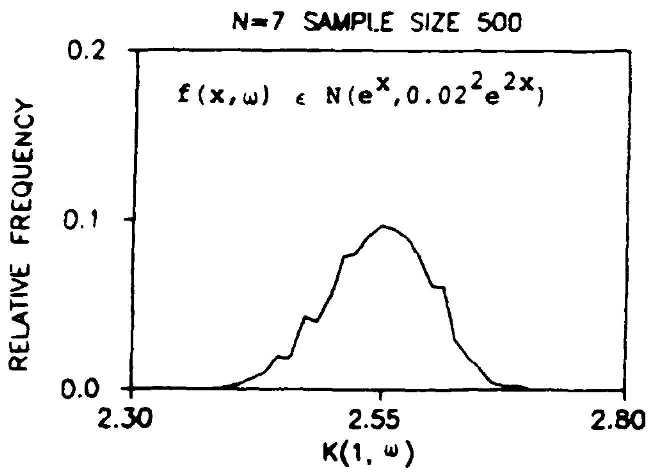


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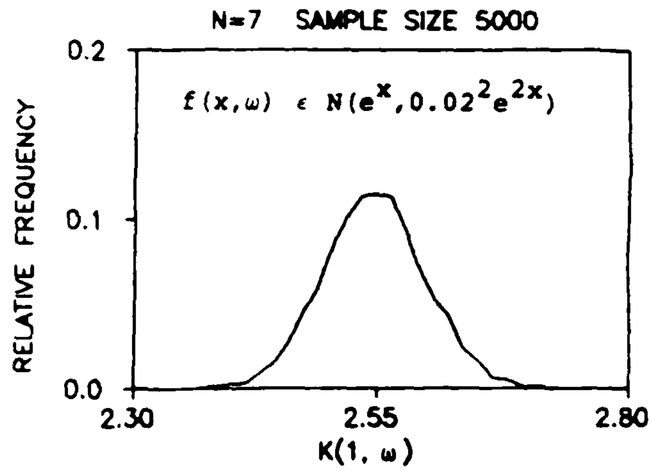


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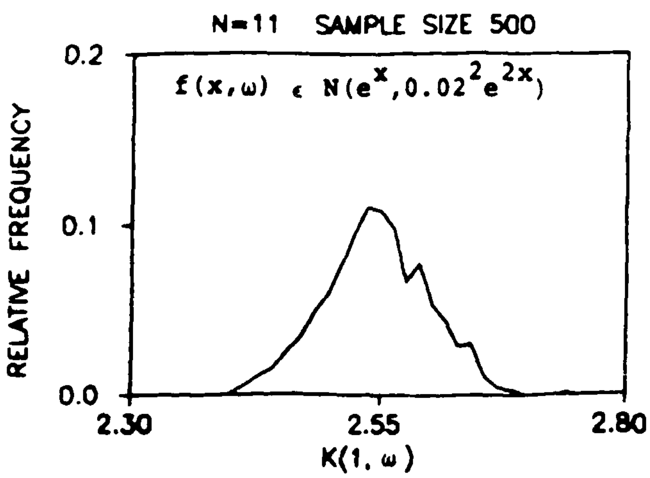


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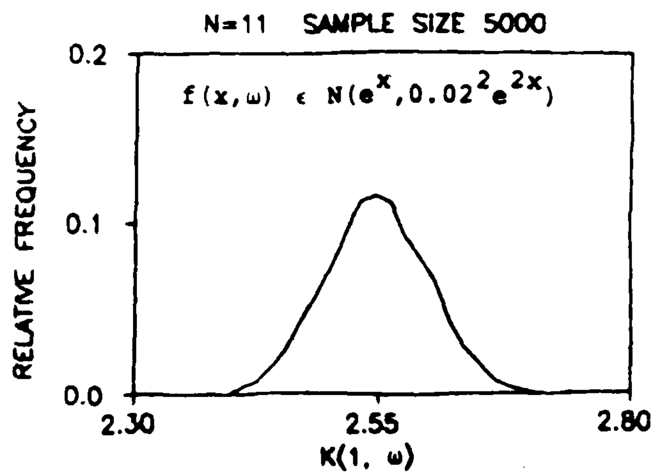


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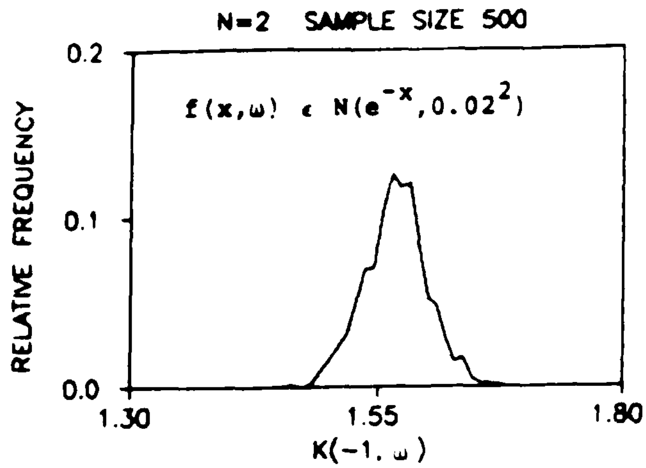


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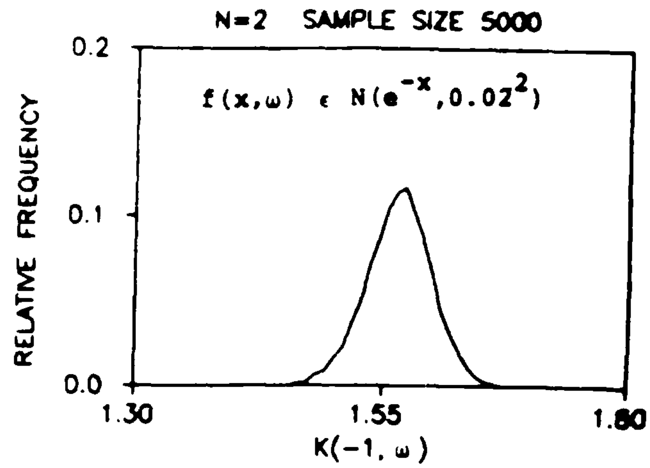


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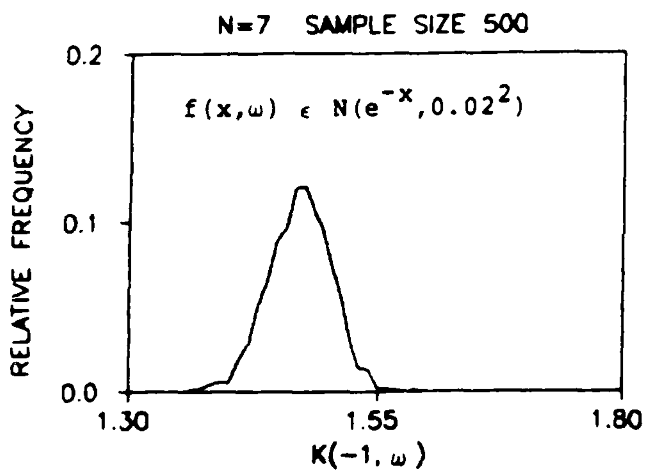


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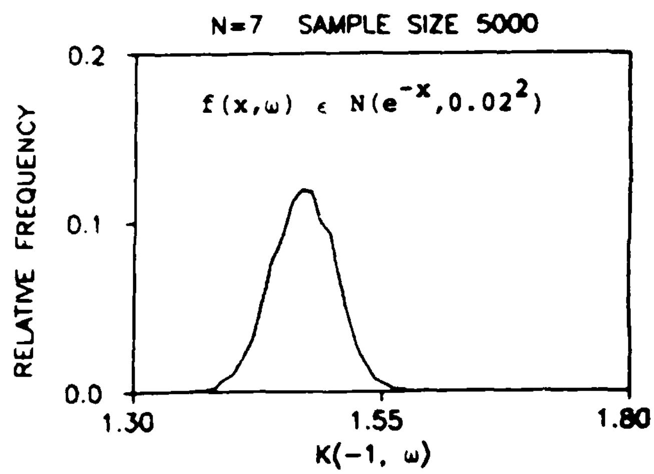


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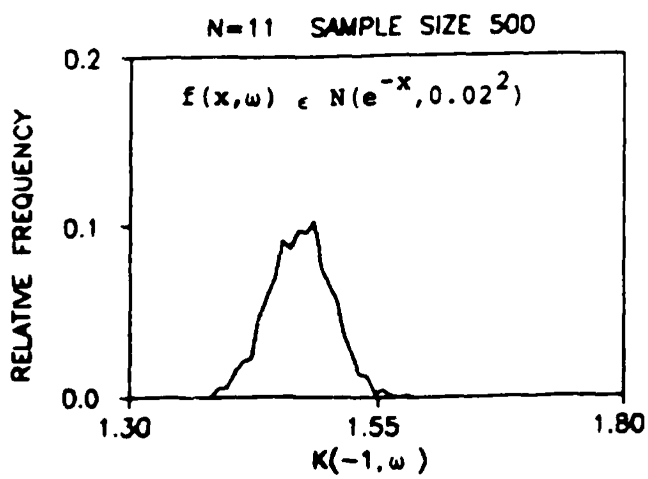


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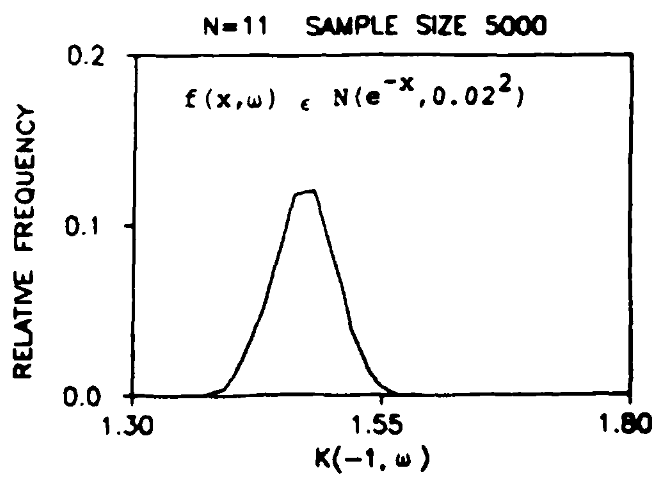


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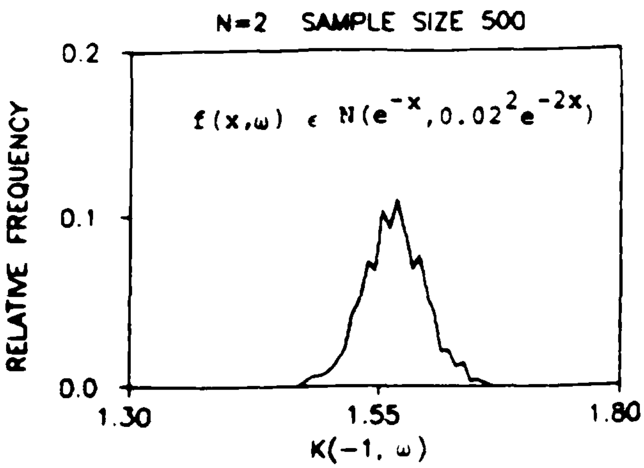


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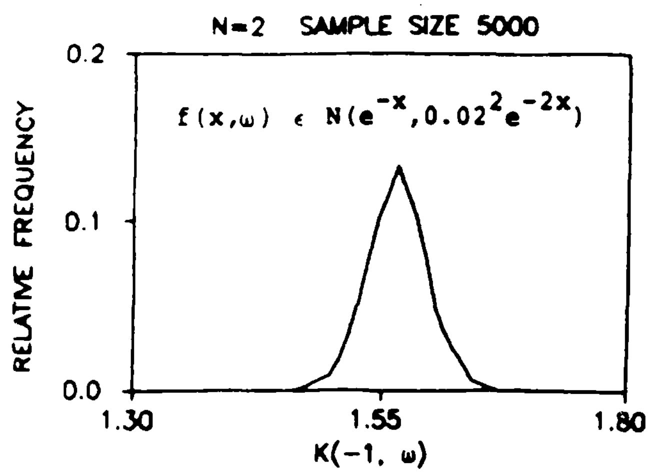


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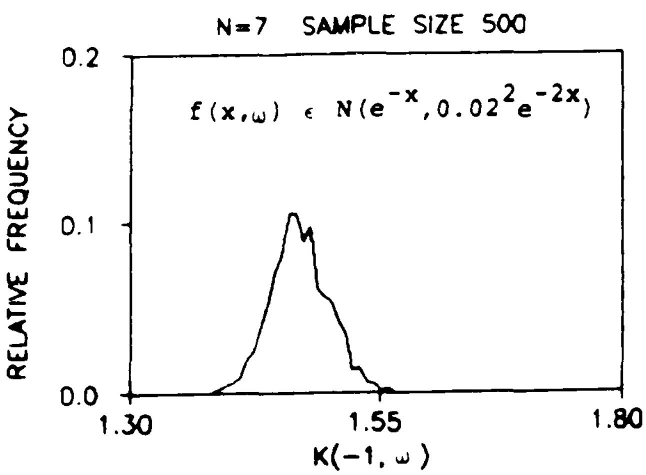


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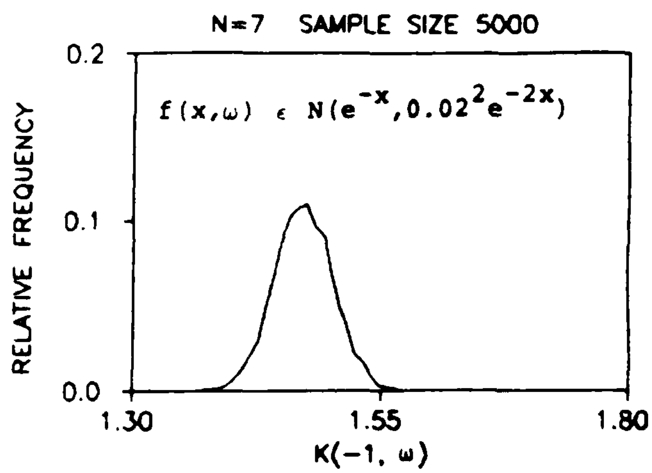


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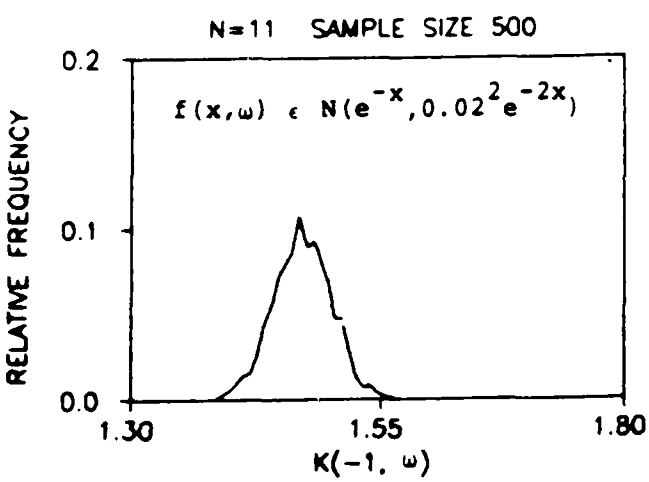


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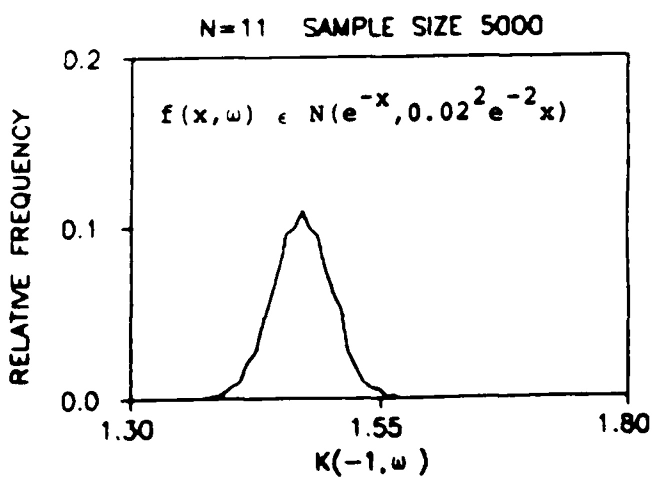


Fig. 30.

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# NUMERICAL SOLUTION OF RANDOM LOVE'S INTEGRAL EQUATION*

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## ABSTRACT

In this paper, we present a method for the numerical solution of random Love's integral equation. The technique involves a Chebyshev series approximation, the coefficients of which are the solutions of a system of random linear equations. The statistical properties of the random solution are evaluated numerically and used to determine the distribution of the random coefficients. We express these distributions in histograms.

algebras $A_L$, L a line of $\pi$. The combinatorial properties of the (0,1) - matrices $\{x_i\}$ which span these algebras are transferred to a polynomial ring $R = k[x_1,...,x_N]/I$, I a certain homogeneous ideal. R is noetherian of Krull dimension n+1 in which certain statements about the prime ideals of R are equivalent to statements about the collineations of $\pi$. The Krull intersection theorem provides an interesting topology for R when $\pi$ admits no collineation.

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#88 (C/P Session 1; Wed. 12:00NOON)

A Family of Algorithms for the Graph Bisection Problem

The Graph Bisection Problem, GBP, is the problem of partitioning the vertex set of a given graph into two subsets of the same size (within one element) so that the number of edges cut is minimized.
We present a family of algorithms for GBP that implements the idea of a restricted branch and bound method.
It is shown that for every constant $\lambda > 0$, there is an algorithm with running time of $O(c^n)$ $(c<2)$, which performs as an exact algorithm on the set of graphs with n vertices and $\leq \lambda n$ edges.
We present the results of computer simulations with the algorithms and discuss several conjectures which are based on these results.

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#89 (C/P Session 8; Thu. 12:00NOON)

APPROXIMATION TO AUTOREGRESIVE MODEL WITH STOCHASTIC COEFFICIENTS

An autoregressive (AR) model with stochastic cofficients is generally modelled by deterministic AR model where stochastic coefficients are replaced by the mean of the stochastic coefficients. It has been proved that this approximation is restrictive and not applicable in general. In this article we suggest suitable improved approximations which are less restrictive and more 'closer' to the exact value. Some applications of power spectral estimation algorithm with stochastic coefficients are also discussed. We illustrate our analytical techniques through computer algorithms and numerical results. These results will highlight the error involved if one uses deterministic AR models insted of stochastic AR models.

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#90 (C/P Session 13; Fri. 5:00PM)

Discriminative Optimization

The extentions to the simplex algorithm required by Goal Programming (GP) inspire a further generalization on a broader class of mixed binary problems with a structure among goal functions.

The Discriminative Optimization (DO) is defined in it's generality and the score of optimization methods like; Set-up cost, Goal Tree Programming, Aggregative Goal Programming, Linear Goal Programming, etc. are shown to be particular cases of the same scheme. Correspondigly, simplex-based solution algorithms for those problems, and even for the Linear Programming, can be obtained as implementations of the general solution process.

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#91 (C/P Session 15; Fri. 5:00PM)

A Model for Cannibalism

Many species of insects and fish practice cannibalism, presumably due to selective pressure toward this behavior. This paper presents a system of non-linear ODE's which reflects population dynamics in an insect species whose members pass through a sequence of stages, and in which later stages cannibalize earlier stages. Solution trajectories must converge when the number of stages is four or less. We conjecture that convergence always obtains but, when

ABSTRACT

APPLIED MATHEMATICS

SEKO, JOSEPH T. M.S. ATLANTA UNIVERSITY, 1986

<u>THE USE OF LAGRANGE MULTIPLIERS AND KUHN-TUCKER'S THEORY IN OPTIMIZATION PROBLEMS</u>

Advisor: Professor Negash Medhin

Thesis dated November, 1986

Lagrange multipliers, penalty methods, and Kuhn-Tucker's theory are some important mathematical tools used in optimization problems. These tools are discussed so that one can appreciate the current areas of optimization research. Moreover, since extensive research work exists for linear optimization problems, only nonlinear applications are discussed.

ABSTRACT

MATHEMATICS

EPHRAIM, DANIEL B.S., MOREHOUSE COLLEGE, 1985

NUMERICAL SOLUTION OF LINEAR INTEGRAL EQUATIONS WITH RANDOM FORCING TERMS

Advisor: M. Sambandham

Thesis dated July, 1987

In Chapter one of this report we define Fredholm integral equations of the second kind, Volterra integral equations of the second kind and differentiate between the two of them and explain why integral equations are important. In Chapter two we discuss numerical procedures to integral equations. The equations we used in this report are of two types: (1) Fredholm equations and (2) Volterra equations. The methods we used for Fredholm equations are: (i) Simpson's rule, (ii) Trapezoidal rule, (iii) Weddle's rule, (iv) the Collocation method, and (iv) the Galerkin method.

For Volterra equations we used the successive approximation method with (i) Simpson's rule, (ii) Trapezoidal rule and (iii) Weddle's rule to evaluate the integrals.

In both Fredholm and Volterra integral equations we have the forcing term to be random. Our simulation results are presented in tables and graphs.

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